

BHATNAGAR INTERNATIONAL SCHOOL

WORKSHEET-CLASS-XII

DIFFERENTIAL EQUATIONS

1Mark Questions

Q.1. Write the order and degree of the following differential equations:

$$(i) \frac{d^2y}{dx^2} + \sqrt{x + \left(\frac{dy}{dx}\right)^2} = 0$$

$$(ii) x \frac{dy}{dx} + \frac{3}{\frac{dy}{dx}} = y^2$$

$$(iii) \frac{\left[2 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = 1$$

$$(iv) \frac{d^4y}{dx^4} + \sin(y''') = 0$$

$$y''' + y^2 + e^{y'} = 0 \quad (v) y'' + 2y' + \sin y = 0$$

$$(vi) y''' + y^2 + e^{y'} = 0$$

$$(vii) \sqrt{1-y^2} dx + y\sqrt{2-x^2} dy = 0$$

$$(viii) x \left(\frac{dy}{dx}\right)^2 + \frac{3}{\frac{dy}{dx}} = y^2$$

$$(ix) px + \sqrt{a^2 p^2 + b^2}, p = \frac{dy}{dx}$$

$$(x) x \frac{dy}{dx} + x \cos^2 \frac{y}{x} = y$$

4-6 Mark Questions

Solve the following differential equations

$$\text{Q.2. } x \frac{dy}{dx} + x \cos^2 \frac{y}{x} = y$$

$$\text{Q.3. } \sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$$

$$\text{Q.4. } e^x \sqrt{1-y^2} dx + \frac{y}{x} dy = 0$$

$$\text{Q.5. } x \cos y dy = (xe^x \log x + e^x) dx$$

$$\text{Q.6. } \sqrt{1+x^2} dy + \sqrt{1+y^2} dx = 0$$

$$\text{Q.7. } \sqrt{1+x^2+y^2+x^2y^2} + xy \frac{dy}{dx} = 0$$

$$\text{Q.8. } \frac{dy}{dx} = \frac{e^x (\sin^2 x + \sin 2x)}{y(2 \log y + 1)}$$

$$\text{Q.9. } \frac{dy}{dx} = \frac{x(2 \log x + 1)}{\sin y + y \cos y}$$

$$\text{Q.10. } xy \frac{dy}{dx} = 1 + x + y + xy \text{ Ans. } y = x + \log x(1 + y) + c$$

$$\text{Q.11. } y(1-x^2) \frac{dy}{dx} = x(1+y^2)$$

Solve the following homogeneous differential equations

$$\text{Q.12. } x^2 dy + y(x+y) dx = 0$$

$$\text{Q.13. } (x^2 - y^2) dx + 2xy dy = 0 \text{ Ans. } (x^2 + y^2) = c|x|$$

$$\text{Q.14. } (x^2 - y^2) dx - 2xy dy = 0 \text{ Ans. } x(x^2 - 3y^2) = c$$

$$\text{Q.15. } \frac{y}{x} \cos\left(\frac{y}{x}\right) dx - \left\{ \frac{x}{y} \sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right) \right\} dy = 0; \text{ Ans. } \left| y \sin\left(\frac{y}{x}\right) \right| = c$$

Q.16. $x \frac{dy}{dx} = y - \tan\left(\frac{y}{x}\right)$ Ans. $\sin\left(\frac{y}{x}\right) = \left(\frac{c}{x}\right)$

Q.17. $xdy - ydx = \sqrt{x^2 + y^2}$ Ans. $y + \sqrt{x^2 + y^2} = cx^2$

Q.18. $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$ Ans. $x + ye^{\frac{x}{y}} = c$

Q.19. $x^2dy + (xy + y^2)dx = 0$ Ans. $x^2y = c(y + 2x)$

Q.20. $xy \log\left(\frac{x}{y}\right)dx + \left\{y^2 - x^2 \log\left(\frac{x}{y}\right)\right\}dy = 0$ Ans. $\frac{x^2}{y^2} \left\{\log\left(\frac{x}{y}\right) - \frac{1}{2}\right\} + \log y^2 = c$

Q.21. $\left[x\sqrt{x^2 + y^2} - y^2\right]dx + xydy = 0$ Ans. $\sqrt{x^2 + y^2} = x \log\left|\frac{C}{x}\right|$

Q.22. $\left(x \cos \frac{y}{x} + y \sin \frac{y}{x}\right)y - \left(y \sin \frac{y}{x} - x \cos \frac{y}{x}\right)x \frac{dy}{dx} = 0$ Ans. $\sec \frac{y}{x} = Cxy$

Q.23. $xe^{\frac{y}{x}} - y \sin \frac{y}{x} + x \frac{dy}{dx} \sin \frac{y}{x} = 0$, Ans. $e^{-\frac{y}{x}} \left\{\sin \frac{y}{x} + \cos \frac{y}{x}\right\} = \log|x|^2 + 1$

Q.24. $x^2ydx - (x^3 + y^3)dy = 0$, Ans. $-\frac{x^3}{3y^3} + \log|y| = c$

Q.25. $(xdy - ydx)y \sin \frac{y}{x} = (ydx + xdy)x \cos \frac{y}{x}$, Ans. $\sec \frac{y}{x} = cxy$

Q.26. Show that the differential equation $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$ is homogeneous and solve it.

Ans. $\sin\left(\frac{y}{x}\right) = \log|Cx|$

Q.27. Show that the differential equation $2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ is homogeneous and find its

particular solution, given that, $x = 0$ when $y = 1$. Ans. $2e^{\frac{x}{y}} + \log|y| = 2$

solve the following linear differential equations

Q.28. $\frac{dy}{dx} - \frac{y}{x} = 2x^2, x > 0$; ans. $y = x^3 + Cx$

Q.29 $\frac{dy}{dx} + \frac{y}{2x} = 3x^2$ Ans. $y\sqrt{x} = \frac{6}{7}x^{\frac{7}{2}} + C, x > 0$

Q.30. $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x, x > 0$ Ans. $y \log x = -\frac{2}{x}(1 + \log x) + C, x > 0$

Q.31. $(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}; x \in (-\infty, -1) \cup (1, \infty)$; Ans.: $y(x^2 - 1) = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + C$

Q.32. $(\tan^{-1} y - x) dy = (1 + y^2) dx$, Ans. $x = (\tan^{-1} y - 1) + ce^{-\tan^{-1} y}$

Q.33. $\frac{dy}{dx} - 2y = \cos 3x$ Ans. $ye^{-2x} = \frac{e^{-2x}}{13} \{3 \sin 3x - 2 \cos 3x\} + c$